## NAME

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## Introduction to hypothesis testing -- The Z-test

## Overview

Goals: This exercise will help you understand the logic and procedures of hypothesis testing. Specifically, the exercise will focus on the logic of hypothesis testing, differences between populations and samples, and the use of probabilities to make statistical decisions.

What do I need to know? To make best use of this exercise, you should know how to use a $z$ table to find probabilities on a normal distribution, and how to calculate the standard error of a mean. Relevant review materials are available from the links below.

What do I need? You will need a table for the standardized normal distribution (z) and a calculator.

Questions: You will be asked several questions. Most questions are multiple choice, you will be given feedback regarding your answers. Attached are an introduction to the applet and pages for recording results of the three sampling exercises. The end of the tutorial contains some "thought" questions; these questions also appear at the end of this packet. Everything in the packet should be completed.

Issues: At many points in this tutorial, you will be dealing with hypothesized means and alternate means. Generally, you would never know the "alternate" mean -- that is, we rarely know the mean of the population from which we sample. You will be given these values however, recall that our conclusions are based solely on our sample results.

You will not need to print any parts of the tutorial. All necessary paper copies are provided. The are instructions on the handout and some on-screen instructions - be sure to attend to both.

Accessing the tutorial: Enter the following address http://wise.cgu.edu Choose Tutorials, Introduction to Hypothesis Testing.

## Hypothesis Testing Applet



You will use features from this applet for the exercise.
The set of curves at the top represent the populations, the red curve (left) represents the null population, the green curve (right) represents the alternative population. When drawing samples, the black boxes in the top set of curves represent the individual scores sampled (Note: $\mathrm{n}=20,20$ boxes appear in upper box). The set of curves at the bottom represent the sampling distributions. When drawing samples, the lower set of curves will place a red arrow where the specific mean falls, a red box remains on the screen representing each sample mean.

Several options exist. First, the box marked alternate mean allows you to choose from a variety of effects or to specific a "custom" distribution. For this exercise we will be using the Mean = 585 , Mean $=625$, and Mean $=667$ options. The box marked $\mathbf{N}$, allows the user to define the sample size. These are the options you will be changing. After changing a value, be sure to click set. Before moving on to a new mean value, click clear.

Population Mean represents the null population mean (i.e., the hypothesized mean). Alternate Mean represents the alternative population mean (the mean of the population we are sampling from). Sigma represents the population standard deviation.

The draw a sample button allows the user to draw a sample using the values specified in the boxes above. The mean, $\mathbf{s}$ (standard deviation), $\mathbf{z}$ value, and $\mathbf{p}$ (probability value) appear for each sample. You will need to record several of these values for your assignment.

## GRE Training Course A

The first GRE training course (A) is very expensive (\$2100 per course). The average score on the GRE quantitative section for graduates of this course is 667 (the population mean for Course A). Recall, the average score for everyone who took the GRE quantitative section without taking a training course beforehand is 555 with a standard deviation of 139 . We will draw several samples of 25 people from the population of students who participated in training course A.

For each sample that you draw, you should record the following information; Sample Mean and Z-score given that the population mean is 555 . Space is provided below.

If Course A did not have any effect, we might expect the sample mean to be about 555. It would be surprising to observe a sample mean much larger than 555 if the sample is drawn from a population with a mean of 555 . The $z$-score for a sample mean tells us the probability of observing a sample mean as big as we found if the population mean for graduates of Course A is really only 555 . The probability is based on the $z$ score for the sample mean compared to a population mean of 555 . A low probability (less than $.05, \underline{z}$ $>1.64$ ) indicates an unlikely result if our sample is drawn from a population with a mean of 555 (i.e., the mean for Course A is the same as the mean for the population).

For this example, choose from pulldown box Alternate Mean $=667$, enter in the box marked $\mathbf{N}=25$. Now click set.

Click on Draw a sample, and record the value shown for Last mean in the first space below, also record the z-score for each value. Then click on Draw a sample nineteen more times and record each mean and z-score.


How many of these 20 sample means would you consider to be "surprising" given the hypothesized population mean was 555 . Consider any result that would occur less than $5 \%$ of the time to be surprising.

If the only data you had were the results of your last sample, would you conclude Course A has a positive effect on GRE scores?

## GRE Training Course B

The second GRE training course (B) is moderately expensive ( $\$ 275$ per course). The average score on the GRE quantitative section for graduates of this course is 625. Recall, the average score for everyone who took the GRE quantitative section without taking a training course beforehand is 555 with a standard deviation of 139 . We will draw several samples of 25 people from the population of students who participated in training course B. You will need a copy of this page before you go choose "go to applet."

For each sample that you draw, you should record the following information; Sample Mean and Z-score given that the population mean is 555 . Space is provided below.

For this example, choose from pulldown box Alternate Mean $=625$, enter in the box marked $\mathbf{N}=25$. Now click set.

Click on Draw a sample, and record the value shown for Last mean in the first space below, also record the $z$-score for each value. Then click on Draw a sample nineteen more times and record each mean and $z$-score.

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How many of these 20 sample means would you consider to be "surprising" given the hypothesized population mean was 555 . Consider any result that would occur less than $5 \%$ of the time to be surprising.

## GRE Training Course C

The third GRE training course (C) is relatively inexpensive (\$50). The average score on the GRE quantitative section for graduates of this course is 585. Recall, the average score for everyone who took the GRE quantitative section without taking a training course beforehand is 555 with a standard deviation of 139 . We will draw several samples of 25 people from the population of students who participated in training course C. You should print this page (or your professor may provide you with a copy) before you go choose "go to applet."

For each sample that you draw, you should record the following information; Sample Mean and Z-score given that the population mean is 555 . Space is provided below.

For this example, choose from pulldown box Alternate Mean = 585, enter in the box marked $\mathbf{N}=25$. Now click set.

Click on Draw a sample, and record the value shown for Last mean in the first space below, also record the $z$-score for each value. Then click on Draw a sample nineteen more times and record each mean and z-score.


How many of these 20 sample means would you consider to be "surprising" given the hypothesized population mean was 555 . Consider any result that would occur less than $5 \%$ of the time to be surprising.

## Follow-Up Questions

The following questions are designed to test your understanding of the material covered in this tutorial. Note: some of these questions provide population information for individual training courses. The standard deviations may differ from the standard deviation for the entire population in previous problems. Be sure you are clear as to what is the population discussed for each question.

## Follow-Up Question 1

Thinking about the results from Training Course C, count the number of times that your sample result would not allow you to reject the null hypothesis.
\# times fail to reject $\qquad$

We know that the true population mean for students in course C is 585 (generally, this information would not be available). Suggest reasons for not being able to reject the null hypothesis when in reality the null hypothesis is false.

Now, compare results from Training Course C to Training Course A. How many times did the sample result fail to allow you to reject the null hypothesis for Training Course A?
\# times fail to reject $\qquad$

Why do these results differ?

## Follow-Up Question 2

Thinking about the three GRE review courses examined in the tutorial, which course do you believe would most reliably produce students who scored above 555 on the quantitative section of the GRE? That is, if you randomly selected 10 students from each course, which would be most likely to score above 555. Why? Assume each program has a standard deviation of 250 .

## Follow-Up Question 3

Given the performance of students in each GRE review course combined with the cost of each course, which course would you prefer (assume you are paying for the class)? Why?

## Follow-Up Question 4

A friend of yours is interested in taking a GRE preparatory class. Your friend says the following "sure, it will cost a lot of money but, I'm going to take Course A. If I take this class I will get a 667 on the quantitative section of the GRE." Assuming that the standard deviation for this course is 250 , what would you tell your friend about the possibility of scoring 667 or better on the quantitative section?

## Follow-Up Question 5

Imagine that you worked for the company offering GRE preparatory course B. Your program has been advertising that the mean score for graduates of the program is 625 (population mean). A former student gets his grades and is very disappointed in his scores on the quantitative section. This student decides to conduct a study of his own. He finds three other individuals who participated in the training course. He finds that the mean score (himself and the three students) was 555 for this section.

Now these four students are in your office. They say "we don't think the program works! None of us scored 625 ! We all scored below 625 , our average score was 555 ! We want our money back!" What can you tell them? Should you give the money back? Do you quit your job because you no longer have faith in the training program? Use Z-scores and statistical reasoning to support your conclusion. (Note: assume that the claim of a population mean of 625 for Course $B$ is accurate and that the $\mathrm{SD}=250$ ).

Would your conclusion differ if you worked for Program A and were faced with the exact same situation? (Note: Assume a population mean of 667 for Course A and that the SD $=250$ ). Why or why not?

